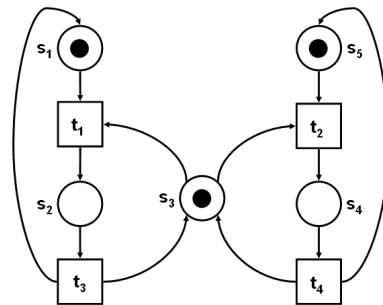


**Formal Foundations of Information Systems**  
**Summerterm 2009**  
 09.07.2009

### 8. Exercise Set: Petri Nets: Invariants and Colored Nets

**Exercise 31 (S-Invarianten und T-Invarianten, 2+1=3 Punkte)**

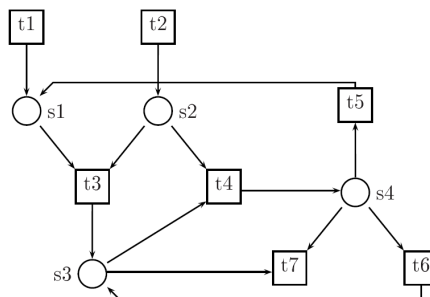
Consider the following Petri Net, which describes a synchronization protocol.



- Compute all  $S$ - and  $T$ -invariants.
- Describe the meaning of these invariants for *this* net.

**Exercise 32 (S-Invarianten und T-Invarianten, 2+1+1=4 Punkte)**

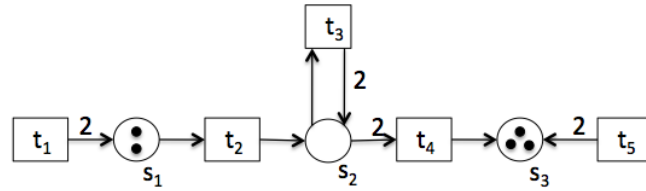
Consider the following Petri Net  $N$ .



- Calculate the  $T$ -invariants (as linear combination of the base vectors).
- Is  $N$  covered by  $T$ -invariants?
- Is  $N$  covered by  $S$ -invariants?

**Exercise 33 (Petri-Netze mit Kapazitäten und S-Invarianten, 2+2=4 Punkte)**

Consider the following Petri-Net (the initial marking is also indicated).

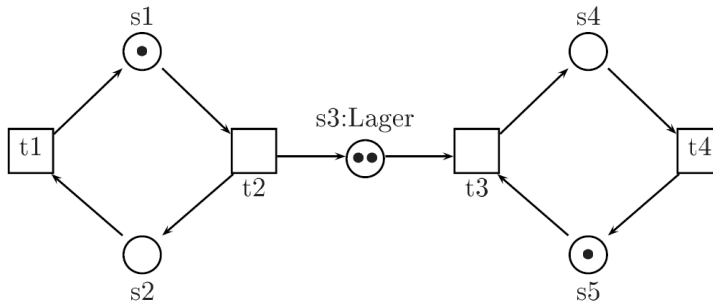


Assume that the petri net has a capacity function  $c$  associated, defined as  $c(s_1) := 6$ ,  $c(s_2) := \omega$ , and  $c(s_3) := 8$  (i.e. place  $s_2$  has unlimited capacity).

- Derive a petri net without capacity that simulates the above net. To this end, follow the discussion from the lecture and introduce two fresh places  $s_1^{co}$ ,  $s_3^{co}$ , and define the associated edges.
- Use  $S$ -invariants to show that the (weighted) sum of all tokens in the two subnets spanned by  $s_1, s_1^{co}$  and  $s_3, s_3^{co}$  in the construction obtained from part (a) is constant.

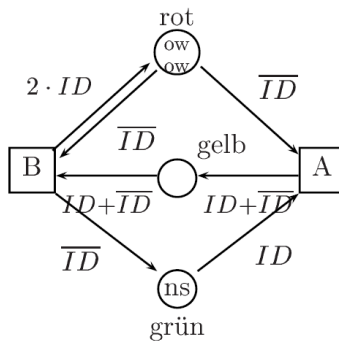
**Exercise 34 (Falten von Petri-Netzen, 2 Punkte)**

Fold the following Petri Net (producer-consumer) such that it has only one place and one transition.

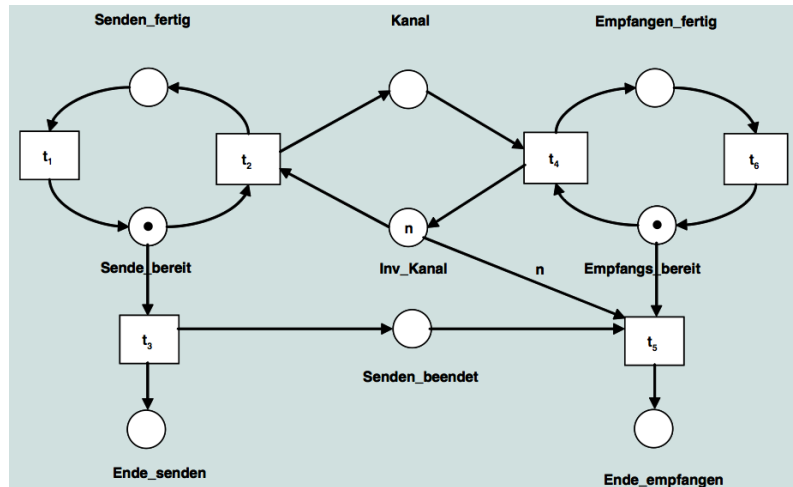


**Exercise 35 (Entfalten von gefärbten Netzen, 2 Punkte)**

Unfold the following Petri Net that models a traffic light into a colored Petri Net, where  $ID$  is the identity function and  $\overline{ID}$  its inverting with respect to ns (north-south) and ew (east-west).



**Exercise 36 (S-Invarianten und T-Invarianten, Bonusaufgabe: 1+1+1+1=4 Punkte)**  
 Consider the P/T-Net



and the following claims.

- a) At most  $n$  messages can be transmitted in parallel.
- b) The sender is either in his final state (*Ende\_senden*), ready (*Senden\_bereit*), or finished (*Senden\_fertig*).
- c) The receiver is either in his final state (*Ende\_empfangen*), ready (*Empfangs\_bereit*), or finished (*Empfangen\_fertig*).
- d) The final state of the receiver can be reached if and only if the channel is empty and the sender has reached his final state.

The lecture slides (p.72–74) contain formalizations for these claims using invariants, as well as a proof sketch. Work out the proofs for these claims, i.e. show that all claims hold.

Due by: 16.07.2009